ECE228B
Spring 2011
Homework 4 Solutions

10.7 Find $\Delta\nu$ for a change in length, Δl

$\nu = c/2nl$

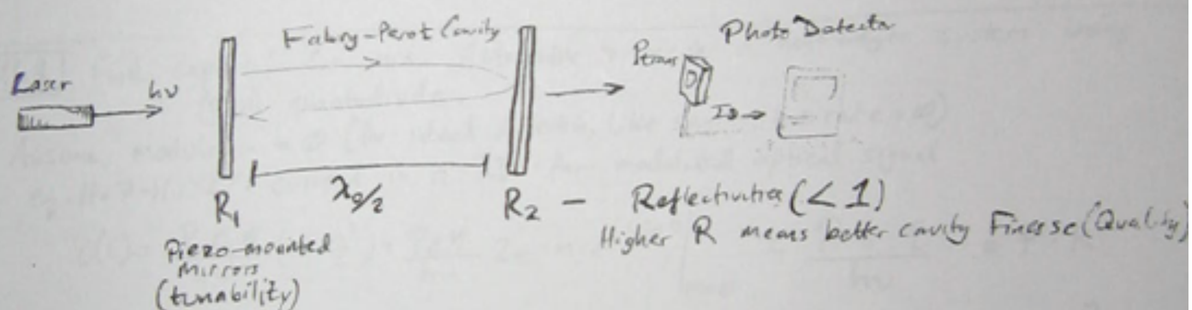
$\nu + \Delta\nu = c/2n(l+\Delta l)$ // $\Delta\nu = \frac{c}{2n(l+\Delta l)} - \nu$

$$\Delta\nu = \frac{c}{2n(l+\Delta l)} - \frac{c}{2nl} = \frac{lc - lc - \Delta lc}{2n(l+\Delta l)l} = \boxed{\frac{-\Delta l \cdot c}{2nl(l+\Delta l)} = \Delta\nu}$$

10.11 There are numerous ways to measure RMS deviation of a laser spectrum

Construct an F-P cavity designed for maximum transmission @ λ_0 (ω_0), and measure $\Delta P_{transmitted}$ over time to find $\Delta\omega$ ($= \omega(t) - \omega_0$)

since $(\Delta\omega)_{RMS} = \langle(\omega(t) - \omega_0)^2\rangle^{1/2}$, use data acquisition software to square the difference, take the time average and square-root the result

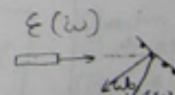


When P_{trans} is minimum, $\lambda_{out} = \lambda_0$ ($\omega_{out} = \omega_0$), higher P_{trans} = deviation

- Determine P_{trans} from I_D
- P_{trans} related to $\lambda(t)$ ($\omega(t)$) via E-M:

$$E_{trans} = E_0 e^{-jkx} - r_1 \cdot r_2 E_0 e^{-jkx+2\theta} \quad // \quad P_{trans} = \frac{|E_{trans}|^2}{\eta}$$

Or: Use a prism: Index of Refraction dep. on frequency: $\varepsilon(\omega)$
or Diffraction Grating, periodic index change on a mirror



both of which essentially create a "Fourier Transform"; Optical frequency converted to spacial position
Detect with PD array (or CCD) & analyze ΔX with relation to $\Delta\omega$.

11.2 Calculate Minimum Received Power in the presence of large optical background power, P_B for a photo conductor.

If noise term is entirely shot-noise limited:

$$i_N^2 = 2q \cdot I \cdot \Delta\nu \quad // \quad I = R \cdot P_{opt} = \frac{\eta q}{h\nu}(P_{sig} + P_B)$$

$$i_{sig}^2 = \left(\frac{\eta q}{h\nu}\right)^2 (P_{sig})^2$$

Signal To Noise Ratio:

$$SNR = i_s^2 / i_N^2 = \frac{R^2 (P_{sig})^2}{2q \cdot R \cdot (P_{sig} + P_B)}$$

[All other noise sources are neglected]

Minimum detectable power is when $i_{sig} = i_N$ // $SNR = 1$

And let $P_B \gg P_{sig}$

$$1 = \frac{R^2 P_{sig}^2}{2q \underbrace{R}_{\eta q/h\nu} \cdot P_B \cdot \Delta\nu} \quad // \quad \boxed{(P_s)_{min} = \left(\frac{2 \cdot P_B \cdot h\nu \cdot \Delta\nu}{\eta}\right)^{\frac{1}{2}}}$$

[typo in question]

11.8 Find expression for min. detectable power in a heterodyne system using a (p-n) photodiode.

Assume modulation = Ø (for ideal detection, like saying, bit rate = Ø)

eq. 11.7-11, Y&Y: current in a P.D. for modulated optical signal

$$i(t) = \frac{P e \eta}{h\nu}\left(1 + \frac{m^2}{2}\right) + \frac{P e \eta}{h\nu} 2e \cdot m \cdot e^{i\omega_m t}\Big|_{M=\emptyset} = \frac{P \cdot e \cdot \eta}{h\nu} = P \cdot R$$

for noise currents, shot noise:

$$i_N^2 = 2q\Delta\nu \cdot I = 2q^2 \frac{P\eta}{h\nu}\Delta\nu$$

[We could have included thermal noise, but it makes no difference to final answer]

Similar to photomultipliers in heterodyne operation (eq. 11.4-8):

$$SNR = \frac{i_s^2}{i_N^2} = \frac{2(P_s \cdot P_L)(e\eta/h\nu)^2}{2q^2(P_s + P_L)\frac{\eta}{h\nu}\Delta\nu}$$

[$(P_s P_L)$ from beating of 'Local' signal with probe Signal, and shot noise is created by $(P_s + P_L)$]

for $(P_s)_{min}$, and $P_L \gg P_s$:

$$SNR = 1 = \frac{P_s \cdot P_L \;(\eta/h\nu)}{P_L \cdot \Delta\nu} \quad // \quad \boxed{(P_s)_{min} = \frac{h\nu \cdot \Delta\nu}{\eta}}$$

[Quantum Limit on shot noise. $(P_s)_{min}\uparrow$ for a modulated signal]

(5) PiN P.D.:

for 8E12 photons, produce 3E12 electrons

A) ► Find η & R for $\lambda_0 = 830\,nm,\ 1300\,nm,\ 1550\,nm$

$$\eta = \frac{\#\text{electrons}}{\#\text{photons}} = \frac{3E12}{8E12} = \frac{3}{8} = \boxed{37.5\%}$$

$$R = \frac{\eta q}{h\nu} = \frac{\eta q}{hc} \cdot \lambda \quad \begin{bmatrix}\text{ignoring} \\ \text{index, } n \\ (n=1)\end{bmatrix} \quad \left(\frac{\text{Amps}}{\text{Watt}}\right)$$

R @ 830nm = 0.25 A/W

R @ 1300nm = 0.39 A/W

R @ 1550nm = 0.47 A/W

B) ► Use Fig 11.15 to find responsivity at $\lambda = 1.3\,\mu m$ for InGaAsP material.

Fig 11.15 is a $\log_2$ scale, so $1.5_{(\mu m)} = 2^{(.378)} \approx 2^{(.4)}$

$$\boxed{\eta(\lambda = 1.3\,\mu m) \simeq 55\%} \quad , R = \eta\left(\frac{q\lambda}{hc}\right) = \boxed{0.577 \left(\frac{A}{W}\right) = R}$$

(6) Si PiN P.D.: $P_{in} = 0\,dBm = 1\,mW$, $\lambda = 0.8\,\mu m$, $BW = \Delta\nu = 20\,MHz$

$\eta = .65$, $i_{dark} = 1\,nA$, $C = 8\,pF$

A) ► Find RMS shot noise current, σ_s^2

$$\sigma_{s\,(\text{shot})}^2 = \langle i_s \rangle^2 = 2q \cdot \Delta\nu\,(I_p + \underbrace{I_d}_{1nA \text{ — negligible compared to } I_p}) \quad // \quad I_p = \overset{(.65)\cdot q\,\lambda/hc\, =\, 0.419\, A/W}{R} \cdot P_{in} = \left(\frac{\eta\, q \cdot \lambda}{hc}\right) \cdot (1\,mW)$$

$$\boxed{\overline{i_{shot}}^2 = 2.69E15\ A^2}$$

$$\text{B) } SNR = \frac{\langle i_{p\,(\text{signal})} \rangle^2}{\langle i_{shot} \rangle^2} = \frac{(P_{in} \cdot R)^2}{2q(P_{in} R)\Delta\nu} = \frac{P_{in} R}{2q\,\Delta\nu} = 65 \times 10^6 \left(\frac{W}{W}\right) = \boxed{78.13\,dB}$$

C) ► for $SNR_{min} = 20\,dB$, find $(P_{in})_{min}$

$$\frac{\langle i_{s\,(\text{signal})} \rangle^2}{\underbrace{\langle i_N \rangle^2}_{\text{noise = shot}}} = \frac{S}{N} = 10^{20dB/10} = 100 \left(\frac{W}{W}\right) \quad // \quad \langle i_s \rangle^2 = (P_{in} \cdot R)^2 = \langle i_N \rangle^2 \cdot 100 \quad // \quad P_{in} = \sqrt{\langle i_N \rangle^2 \cdot 100}/R$$

$$P_{in} = \sqrt{\langle i_N \rangle^2 \cdot 100}/R = \sqrt{\cancel{(2.69E{-15}) \cdot 100}}/0.419\tfrac{A}{W} = \boxed{\cancel{1.24}\ \mu W}$$

$$\langle i_{Noise} \rangle^2 = 2q \cdot \Delta\nu\,(P_{in} \cdot R)$$

$$P_{in} \cdot P_{in}^{-1/2} = P_{in}^{1/2} = \sqrt{2q \cdot \Delta\nu \cdot 100 \cdot R}/R \quad // \quad (P_{in})_{min} = 2q\,\Delta\nu \cdot 100/(.419)$$